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Analysis of turbulence energy spectrum by using particle image velocimetry

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Abstract

Transport phenomena occur frequently in industrial problems. Most of the turbulent transport properties can be directly associated with the turbulent energy dissipation rate; hence it is a very significant parameter in the design of chemical processing equipment. To develop a better chemical processing equipment design, a thorough knowledge of the effect flow structure on local turbulence parameters like turbulent kinetic energy, eddy diffusivity and the energy dissipation rate are required. Turbulence is heterogeneous in most of the process equipment. Hence, the use of spatial average energy dissipation rate causes error in modelling of turbulent transport processes. In this present work, particle image velocimetry (PIV) is used to obtain the energy spectrum from grid generated homogeneous turbulence velocity data. The model of energy spectrum given by Kang et al. (2003) has been fitted to this energy spectrum using energy dissipation rate. A different approach, based on a third order structure function and velocity gradient technique has been used to compute the energy dissipation rate. The model predictions have been verified by experimental PIV velocity data from oscillating grid apparatus.

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1. Main text

The majority of indirect methods used to calculate energy dissipation rate, ε , require velocity data. The four main approaches include equation based on dimensional analysis, equation based on spatial velocity derivative, fitting a model spectrum to experimental energy spectrum and use of Karman-Howarth equation to fit experimental structure

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function data. Each method involves some approximation based on theory of turbulence. The selection of a particular method to calculate ϵ depends on the validity of assumptions made by that particular method for the flow situation under consideration. Sometimes, it would be necessary to estimate ϵ using more than one method to cross check the value predicted by either method. Such parallel estimation also facilitates calculation of uncertainty in the value of ϵ calculated.

To assist with such a selection, it was thought desirable to put all the four methods to test on velocity data from an apparatus where the flow field represents homogeneous and isotropic turbulence. Whilst a few publications in published literature describe estimation of turbulent energy dissipation rate in geometries like stirred tank Gabriele et al. [3] and submerged jet Deshpande et al. [1]; there are very few papers which test different theories to calculate energy dissipation rate in geometry with controlled, homogeneous and isotropic turbulence. One such geometry which facilitates experimentalists to generate homogeneous and isotropic turbulence is oscillating grids Doroodchiet al. [2]. It has zero net mass flow through the system. Also, the turbulence intensity and energy dissipation rate can be easily controlled by changing the oscillation frequency of the grid. Considering these advantages, oscillating grid assembly is used in the present work as a test case.

All of the methods to calculate ϵ described above require velocity data. Particle image velocimetry allows measurement of 2D velocity vector field over a plane in the flow domain. It is possible to calculate important quantities like integral length scale, velocity gradient, structure functions and spatial energy spectrum from PIV data. Therefore, in the present work we use PIV to get 2D velocity field in the oscillating grid apparatus. In summary, there are four different methods to calculate ϵ from velocity data. Each one of them has limitations depending on the assumptions in underlying theory. In the present work, we put these four methods to test on the 2D velocity data measured by PIV in an oscillating apparatus, which generates isotropic and homogeneous turbulence. Specific aims of the present work are:

- Obtain 2D PIV velocity data for different oscillating grid conditions (with different energy dissipation rate).
- Compute the time averaged 1D energy spectrum for flow fluid to obtain the 3D energy spectrum, which is then fitted using the relationship of Kang et al. [6] to determine the specific energy dissipation rate.
- Apply 3rd order structure function analysis to the 2D velocity fluctuation data to determine the energy dissipation rate.
- Estimate energy dissipation rate directly from spatial velocity derivative.
- Compare both predictions for specific energy dissipation rate with that estimated by direct analysis (eq. 3) of the experimental data.
- Comment on the merits for each of the methodologies for determination of ϵ .

2. Experimental Section

The flow system shown in Fig. 1 has been described in Hoque et al. [5] and was the same as that used previously by Doroodchiet al. [2]. It consisted of oscillating grids in a glass tank filled with water. The width of tank was 300 mm. The grids size was 150 mm×150 mm and the grids were oriented vertically. The tank was supported by a steel bench with adjustable height. The grids were made of aluminum with bar thickness of 6mm and 30mm spacing between the bars. The overall open area of grid was approximately 64%. Stepper motors were used to drive the connected through eccentric cams to generate grid oscillation. The stroke length of the oscillation was 18 mm, while the separation between two grids was 110 mm. The cranks were adjusted to have 180° phase lag, causing grids to move towards each other during first half of the cycle and away from each other during the other half cycle. The stroke length and the grid separation were adjusted to match those used by Doroodchiet al. [2].

Fluorescent polystyrene-DVB particles (Kanomax, Japan) with 30 micron mean diameter were used as PIV tracer particles. Tests were conducted with grid oscillation frequencies of 2.5 and 4 Hz, respectively. The stroke length and the distance between the grids were kept the same for both the runs. A high speed digital PIV technique was employed in this study to measure the longitudinal (x component) and transverse (y component) velocity of water. Phantom v640 camera was used with resolution of 1600×1600 pixels. The image recording frequency was 400 Hz. 4000 images were recorded in each PIV run. PIV processing was done with Dantec Dynamic studio software. The measurement plane was located at the center of the tank. Field of view was 60 mm × 60 mm, and vector spacing was 0.6 mm.

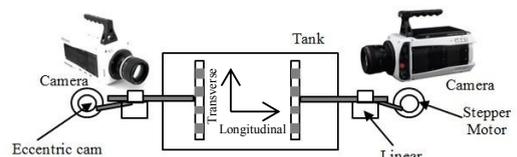


Fig. 1: Oscillated grid generated turbulence

3. Mathematical Model for Calculating Energy Dissipation Rate

Theoretically, ε can be calculated from an exact expression in terms of viscous stress by Hinze[4]. For a three dimensional flow field, ε is given by the following equation:

$$\varepsilon = \nu \left(2 \left(\frac{\partial u_1}{\partial x_1} \right)^2 + \left(\frac{\partial u_2}{\partial x_1} \right)^2 + \left(\frac{\partial u_3}{\partial x_1} \right)^2 + \left(\frac{\partial u_1}{\partial x_2} \right)^2 + 2 \left(\frac{\partial u_2}{\partial x_2} \right)^2 + \left(\frac{\partial u_3}{\partial x_2} \right)^2 + \left(\frac{\partial u_1}{\partial x_3} \right)^2 + \left(\frac{\partial u_2}{\partial x_3} \right)^2 + 2 \left(\frac{\partial u_3}{\partial x_3} \right)^2 + 2 \left(\frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \right) + 2 \left(\frac{\partial u_1}{\partial x_3} \frac{\partial u_3}{\partial x_1} \right) + 2 \left(\frac{\partial u_2}{\partial x_3} \frac{\partial u_3}{\partial x_2} \right) \right) \quad (1)$$

In case of 2D PIV measurements, we have only two components of velocity measured in two directions, which yield four components of velocity gradient tensor out of 9 required by the above equation. In order to calculate ε using four components of velocity gradient tensor, above equation is modified using the assumption of isotropy as follows:

$$\varepsilon = \nu \left(3 \left\{ \left(\frac{\partial u_1}{\partial x_1} \right)^2 + \left(\frac{\partial u_2}{\partial x_2} \right)^2 \right\} + 3 \left\{ \left(\frac{\partial u_1}{\partial x_2} \right)^2 + \left(\frac{\partial u_2}{\partial x_1} \right)^2 \right\} + 6 \left(\frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \right) \right) \quad (2)$$

The accuracy of ε is hampered by the fact that we approximated 5 components of velocity gradient tensor with the isotropy assumption. Besides the isotropy assumption, the numerical error in estimation of the velocity gradient from PIV data also adds to the uncertainty in the value of ε .

An alternative to calculation of ε from velocity gradients is the following equation based on the dimensional analysis Hinze[4]:

$$\varepsilon \propto \frac{u_{rms}^3}{l} \Rightarrow \varepsilon = k \frac{u_{rms}^3}{l} \quad (3)$$

Where u_{rms} is the RMS fluctuating velocity, l is the integral length scale and k is the proportionality constant. The value of k varies between 0.5-2 by White et al. [9]. ε can be calculated using eq. (3) by assuming a value of k suitable for the nature of flow under consideration. In the current work, we are using $k=1$ to calculate ε .

Above methods facilitate direct calculation of ε from velocity data. Although these methods are straightforward to implement, there is a level of uncertainty in the value of ε estimated using them. As an alternative, the methods based on theory of turbulence can be used. Two such methods were used in the current work. First one is based on the energy spectrum and second one uses the Karman-Howarth equation of structure function transport. Both of these methods are statistical and are less susceptible to noise.

3.1. ε from Energy Spectrum

Energy spectrum is the distribution of turbulent kinetic energy between eddies of different size. Eddy size is represented by its wavenumber ($\kappa = 2\pi/\lambda$, λ is the wavelength of eddy). Eddy energy for respective wavenumber is calculated using FFT of velocity data. The most popular result about energy spectrum is the Kolmogorov -5/3 power law fits the spectrum for the inertial subrange. It is expressed as:

$$E(\kappa) = C \varepsilon^{2/3} \kappa^{-5/3} \quad (4)$$

Where C is the Kolmogorov constant and its value is 1. If the values of wavenumber κ and $E(\kappa)$ are known, ε can be calculated using eq. 4. $E(\kappa)$ is the three dimensional (3D) energy spectrum, which represents the integral of eddy energies along a sphere of radius κ in wavenumber space at the measurement point. Experimentally, the energy spectrum is calculated by taking FFT of the velocity space or time series. Such a spectrum is one dimensional (1D) in wavenumber space since the Fourier transform is applied in one direction only. Hence, eq. 4 cannot be directly

fitted to experimental spectrum to calculate ε . 1D spectrum can be calculated from 3D spectrum by using the following equation:

$$E_{11}(\kappa_1) = \int_{\kappa_1}^{\infty} \frac{E(\kappa)}{\kappa} \left(1 - \frac{\kappa_1^2}{\kappa^2}\right) d\kappa \tag{5}$$

Where $E(\kappa)$ is the 3D energy spectrum function, κ is the resultant wavenumber (of wave number components in three directions: $\kappa_1, \kappa_2, \kappa_3$), κ_1 is the component of wavenumber in principal flow direction. $E(\kappa)$ can be Kolmogorov’s power law spectrum as in eq. 4. However, eq. 4 is valid only for a small range of κ called the inertial subrange. The following model for $E(\kappa)$ proposed by Kang et al. [6]:

$$E(\kappa) = C\varepsilon^{2/3}\kappa^{-5/3} \left[\frac{\kappa l}{[(\kappa l)^{\alpha_2 + \alpha_1}]^{1/\alpha_2}} \right]^{5 + \alpha_3} e^{-\alpha_4 \kappa \eta} \times \left[1 + \alpha_5 \left(\frac{1}{\pi} \arctan\{\alpha_6 \log_{10}(\kappa \eta) + \alpha_7\} + \frac{1}{2} \right) \right] \tag{6}$$

Where α_i ($i = 1$ to 7) are adjustable parameters. The values of α_i and ε are adjusted such that the model spectrum fits the experimental spectrum. The steps of curve fitting are as follows:

1. Experimental $E_{11}(\kappa)$ is calculated by taking FFT of space series of u velocity in x direction from PIV data. It is averaged over each PIV vector field and over 4000 such vector fields.
2. Model $E(\kappa)$ is calculated from eq. 6 with initial guess values of α_i and ε .
3. Model $E_{11}(\kappa)$ is calculated using $E(\kappa)$ from step 2 and error in estimation of $E_{11}(\kappa)$ is calculated.
4. Values of α_i and ε are changed and step 2 and 3 are repeated till the model $E_{11}(\kappa)$ curve fits the experimental $E_{11}(\kappa)$.

3.2. ε from Karman-Howarth equation

Another approach to calculation of ε with strong theoretical grounds is using Karman-Howarth equation (Lindborg, [7]; Pope,[8]; Kang et al. [6]). The Karman-Howarth equation is:

$$\frac{3}{r^5} \int_0^r r^4 \frac{\partial}{\partial t} D_{uu}(r, t) dr = 6\nu \frac{\partial D_{uu}}{\partial r} - D_{uuu} - \frac{4}{5} \varepsilon r \tag{7}$$

where $D_{uu}(r, t)$ and $D_{uuu}(r, t)$ are second and third order structure functions, defined as:

$$D_{uu}(r, t) = \langle [u_1(x + e_1 r, t) - u_1(x, t)]^2 \rangle \tag{8}$$

$$D_{uuu}(r, t) = \langle [u_1(x + e_1 r, t) - u_1(x, t)]^3 \rangle \tag{9}$$

Kolmogorov’s second similarity hypothesis $\partial D_{uu} / \partial t = C\varepsilon^{2/3} r^{2/3}$ and the $\kappa - \varepsilon$ model for the decay of the dissipation $\partial \varepsilon / \partial t = -C_{\varepsilon 2} \varepsilon^2 / \kappa$ can be used to calculate the time dependent term $(\partial D_{uu}(s, t) / \partial t)$. Substituting these two expressions in eq.7, after considerable manipulations we get the following equation:

$$-\frac{D_{uuu}}{r\varepsilon} = \frac{4}{5} - C_1 \left[4 \left(\frac{r}{\eta} \right)^{-4/3} + \frac{4\sqrt{15}C_{\varepsilon 2}}{17} Re_{\lambda} \left(\frac{r}{\eta} \right)^{2/3} \right] \tag{10}$$

Here, $Re_{\lambda} = (\nu / u_{rms} \lambda)$. And $\lambda = (15\nu u_{rms}^2 / \varepsilon)^{1/2}$ is Taylor microscale. Also, according to literature the values of C and $C_{\varepsilon 2}$ were varied in the range 1.6-2.3 and 1.6-2.1, respectively. The steps to calculate using eq. 10 are as follows:

1. Experimental D_{uuu} values for different vector spacing ‘ r ’ are calculated using the PIV vector field. The vector field was 100×100 vectors. r was varied from 1 to 50 to satisfy Nyquist sampling criteria. u_{rms} was also calculated from PIV dataset.
2. Using an initial guess of ε , λ and η were calculated. The model values of D_{uuu} as a function of r were calculated from eq. 10 using initial guess values of ε , C_1 , $C_{\varepsilon 2}$.

3. The model D_{uuu} was compared with experimental D_{uuu} and the error in estimation was calculated.
4. Steps 2 and 3 were repeated until the error in estimation of D_{uuu} was minimized.

4. Results and Discussion

To establish the turbulence in the flow field in the current experiment is homogeneous and isotropic, we compare the energy spectra in both the longitudinal (x) and transverse (y) directions. Fig. 2 shows such spectra for grid oscillation frequency of 4 Hz. Fourier transform was used to evaluate the energy of eddies as a function of frequency. The wavenumber was calculated using $\kappa = 2\pi/\lambda$. Both the longitudinal and transverse energy spectra follow similar trend in the inertial sub range and overlap within experimental accuracy. Such overlap highlights the fact that velocity field in the current experimental system represents nearly homogeneous and isotropic turbulence.

In another approach we used the model energy spectrum given by Kang et al. (2003) with the procedure described in

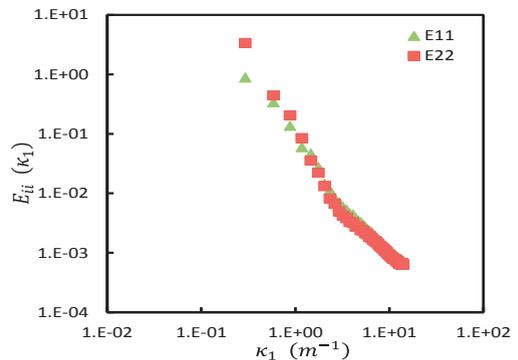


Fig. 2. Comparison of the longitudinal and transverse energy spectrum for grid generated turbulence system ($i = 1,2$).

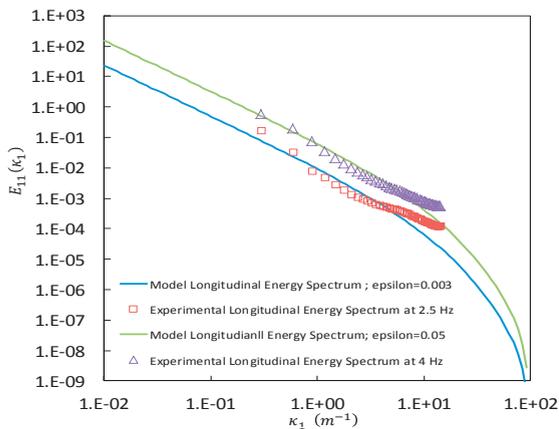


Fig. 3. Longitudinal energy spectra (solid lines) with the experimental data at different frequency using Kang et al. (2003) model equation.

rate. Fig. 3 shows the fitted energy spectrum for grid oscillation frequency of 2.5 and 4 Hz. The optimum curve fit parameters are: $C = 1.6, \alpha_1 = 0.4, \alpha_2 = 1.0, \alpha_3 = 4.0, \alpha_4 = 20, \alpha_5 = 0.522, \alpha_6 = 10.0$ and $\alpha_7 = 12.58$. The energy dissipation rate was computed as 0.003 and 0.050 for the grid frequency of 2.5 and 4 Hz respectively.

The third order structure function was calculated from PIV data. The model curve in eq. 10 was fitted to the experimental data using procedure described in section 3.2. Fig. 4 shows the plot of $D_{uuu}/r\epsilon$, using the value of ϵ determined by curve fitting. The peak value of $D_{uuu}/r\epsilon$ is close to 4/5 in accordance with Kolmogorov's similarity hypothesis. The peak values are below 0.8 (0.57 and 0.78 respectively). The energy dissipation rate was computed as 0.001 and 0.020 for the grid frequency of 2.5 and 5 Hz respectively.

The histogram of energy dissipation rate calculated using the velocity gradient method for the grid oscillating frequency of 2.5 and 4 Hz has been plotted in Fig. 5. The mean energy dissipation rate was found to be 0.0014 and 0.0046 respectively. The value is close to the one estimated using RMS velocity and integral length scale. While the results for the present case are good, it should be noted that this method is far more sensitive to noise in PIV data than the energy spectrum or structure function method. The spatial resolution of PIV should be of the order of

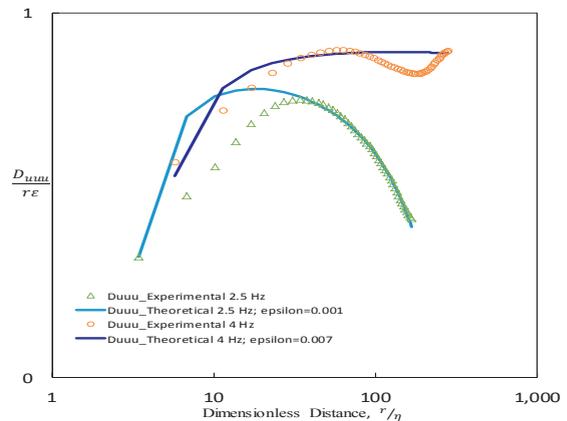


Fig. 4. Karman-Howarth Model to calculate energy dissipation for PIV data.

Kolmogorov length scales for the velocity gradient method to give good estimates, which is not practical in every case. To get further insight into the merit of energy spectrum model for the present case, the 1D energy spectrum in the transverse (y) direction was calculated using same curve fit parameters and ϵ values as those used to plot Fig. 3. The equation to calculate $E_{22}(\kappa_1)$ from $E_{11}(\kappa_1)$ is as follows:

$$E_{22}(\kappa_1) = \frac{1}{2} \left[E_{11}(\kappa_1) - \kappa_1 \frac{dE_{11}(\kappa_1)}{d\kappa_1} \right] \tag{11}$$

$E_{22}(\kappa_1)$ is plotted in Fig. 6 along with experimental $E_{22}(\kappa_1)$ data. The experimental and model spectra match for both of the grid oscillation frequencies. Such agreement highlights the strength of the model energy spectrum as it

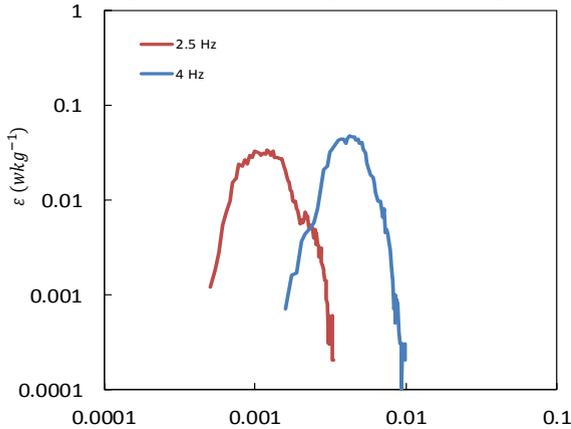


Fig. 5. Calculating Energy dissipating using velocity gradient field.

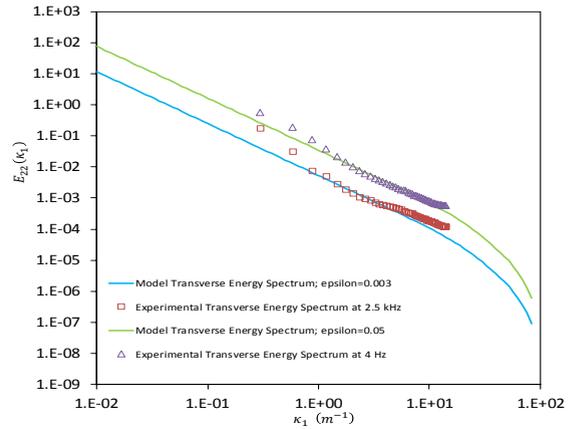


Fig. 6. Comparison of the transverse energy spectra (solid lines) with the experimental data using Kang et al. (2003) model.

describes the underlying physics properly. Hoque et al. [5] used the Pope [8] model with the same method described in section 3.1 but could not get a similar fit to experimental data for both the $E_{22}(\kappa_1)$ from $E_{11}(\kappa_1)$ using same curve fit parameters. It would seem, therefore, that the model spectrum by Kang et al. [6] provides superior results for prediction of energy spectrum.

The specific energy dissipation rate values calculated by all four methods for different grid frequencies are reported in Table 1. The predicted value of ϵ is different for each method. The ϵ values predicted by RMS velocity, velocity gradients and structure functions are comparable. The ϵ values predicted using the energy spectrum is higher compared to those for the other three

Table 1. Comparison table for energy dissipation rate

Frequency, f (Hz)	r.m.s. velocity, u_{rms} (mm/s)	Integral length scale, l (mm)	Epsilon, (ϵ), Experimental Value (Approx.) w/kg	Epsilon, (ϵ), Velocity Gradient Method; w/kg	Epsilon (ϵ), Karman-Howarth Method; w/kg	Epsilon, (ϵ), Functional Method ; w/kg
2.5	9.60	1.31	0.0007	0.0014	0.0010	0.0030
4	22.70	1.01	0.0115	0.0046	0.0070	0.0500
5	29.23	1.02	0.0244	0.0109	0.0200	0.0600
6.25	37.88	1.13	0.0482	0.0341	0.0300	0.2000

The deviation of predicted ϵ values for the spectrum might be attributed to the numerical technique to evaluate experimental energy spectrum and the limitation on the number of images used for time averaging the spectrum

5. Conclusions

The energy dissipation rate in the grid generated turbulence calculated using RMS velocity and integral length scale, Velocity gradients, Energy spectrum and Karman-Howarth equation. The values of ϵ predicted by different methods were found to be in agreement with one another. The spectral slope of -5/3 was found to be present in the energy spectrum in the inertial sub range. The experimental longitudinal and transverse spectra for both velocity

components overlap, indicating homogeneous and isotropic turbulence. Also, the peak value of $D_{uuu}/r\epsilon$ was found to be close to 4/5 confirming Kolmogorov's similarity hypothesis. For same local energy dissipation rate the longitudinal and transverse energy spectrum from Kang et al. [6] model gives better result than original Pope [8] model.

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